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**A COLLECTIVE MODEL FOR
ELECTRON SCATTERING FROM NUCLEI**

RANDALL CLAY STEPHENS

A COLLECTIVE MODEL FOR
ELECTRON SCATTERING FROM NUCLEI

by

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ABSTRACT

The work of Lewis and Walecka on the structure of the giant resonance in Carbon 12 and Oxygen 16 has seemed to indicate that the single-particle, shell-model theory gives better results than collective models -- in particular the Goldhaber - Teller and Steinwedel - Jensen models. The transverse form factor as a function of momentum transfer is observed experimentally to fall to a minimum and rise again in the range from zero to 0.6 f^{-1} . This dip is predicted by the Brown particle-hole theory model but is not predicted by either of these collective models. The purpose of this paper is to develop a collective model which could possibly give this dip in the form factor. Further work is necessary to determine if this model can actually do so. A semiclassical treatment is used to find the matrix elements for the assumed model. The transverse form factor is then calculated for two nuclear charge density functions -- constant density and the Fermi distribution.

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1. Introduction.

Electron scattering is a very useful tool for the study of nuclear structure, since the cross-section for electron scattering from a nucleus can be related to the basic geometric details of the structure of the nucleus. The incoming electron is assumed to interact with the nucleus by means of the electromagnetic field. This interaction is known, whereas the nuclear interaction (strong force) is not.

The parameters of electron scattering are the initial momentum \vec{k} and initial energy E of the incoming electron, which are controlled by the experimenter; and the final momentum \vec{k}' and the final energy E' , which are measured. The cross-section can be found as a function of the momentum transfer $\vec{q} \equiv \vec{k} - \vec{k}'$ from the electron to the nucleus and of the excitation energy, which can be found from E, E' , the scattering angle, and considerations of the nuclear recoil. One can then vary \vec{q} for a fixed excitation energy (i. e., for an excitation of the nucleus to a particular level), and get the cross-section as a function of q (where $q = |\vec{q}|$) for that level. This cross-section is related to the Fourier transform of the charge and current densities of the nucleus. Thus, we can get information about the charge and current densities of the nucleus from electron scattering.

A so-called "dipole state" is a feature of the electron scattering cross-section for most nuclei. The dipole state is a state strongly

excited by photons of the appropriate energy (usually 10 - 25 Mev). It shows up as the "giant dipole resonance", which is a resonance peak in the cross-section as a function of energy. The dipole state is so named because it has a large electric dipole matrix element with the ground state. As a result of this last fact, these states lead to strong photon absorption cross-sections which are orders of magnitude larger than those of other multipoles.

If nuclear recoil is ignored, the cross-section for electron scattering can be written in the general form [2] :

$$\frac{d\sigma}{d\omega} = \frac{8\pi\alpha^2}{q^4} \frac{k'}{k} \left[\frac{q^4}{q^4} 2kk' \cos^2(\theta/2) f_L^2(q) + \frac{2kk'}{2} \sin^2(\theta/2) [q^2 + 2kk' \cos^2(\theta/2)] f_t^2(q) \right]$$

where \underline{q} is the four-momentum transfer and θ is the angle between the incoming and final electron momenta. f_L and f_t are the longitudinal and transverse form factors. Regardless of the model used, the cross-section will have this form and will depend on these two form factors. If nuclear recoil is taken into account, the cross-section is simply multiplied by a factor

$$\frac{1}{1 + \frac{k' - k \cos \theta}{E'}}$$

The transverse form factor can be easily separated out, since for $\theta = 180^\circ$, the coefficient of the longitudinal form factor goes to zero.

Figure 1 shows the experimental results for the square of the transverse form factor for electron scattering from C^{12} as a function of momentum transfer, along with the predictions of two collective models (the Goldhaber - Teller and Steinwedel - Jensen models) and two particle-hole shell models (the Brown and Gillet - Melkanoff models). Figure 2 shows the experimental results for the transverse form factor for O^{16} and the predictions of the Brown and Goldhaber - Teller models. For both C^{12} and O^{16} , the observed behavior is more closely predicted by the shell models than by either of these two collective models. Lewis and Walecka assert that this is conclusive proof that the collective mode description of these excited states is not adequate, and that the particle-hole shell model description is correct. The purpose of this paper is to develop a collective model which can predict the observed behavior of the form factor.

In the Goldhaber - Teller model, the neutrons of the excited nucleus are assumed to move against the protons as a unit, thus giving the effect of an harmonic oscillator, with the restoring force being due to changes in the overlap of the protons and the neutrons [2]. The interaction Hamiltonian is assumed to be

$$H = \frac{1}{2} P^2 / u + \frac{1}{2} uwQ^2 ,$$

where w is the frequency of oscillation, $u = NZM / A$ is the reduced mass of the system, \bar{Q} is the coordinate representing the separation of the center of mass of the proton system from the center of mass of

the neutron system, and \bar{P} is the canonical momentum associated with the coordinate \bar{Q} . P and Q are quantized and the displacements are assumed to be small. The charge and current densities are found from the motion of the protons.

In the shell model theories, a single nucleon is assumed to be excited from a closed shell by the interaction with the incoming electron. This nucleon then interacts with the resulting "hole" left in the shell it previously occupied. The nucleon and the hole are treated as point particles. The excited state of the nucleus is assumed to be a linear combination of the single-particle excitations, and the charge, current, and magnetization matrix elements between the single-particle states are found.

In this paper, a collective mode is assumed and a semiclassical treatment is used to find the matrix elements of the Hamiltonian describing the interaction between the electron and the nucleus. The cross-section is then found to depend on the two expected form factors. These form factors are then calculated for a nucleus of constant density and for a nucleus whose charge distribution is the Fermi distribution.

2. The Model.

The collective mode assumed is an harmonic oscillation of the protons and neutrons in the θ -direction about some axis, so that the charge density of the excited part of the nucleus is:

$$\rho(r) = Z'e f(r) (1 + a \cos \theta e^{i\omega t})$$

where Z' is the number of protons involved in the excitation, and $Z'e f(r)$ is the ground state charge density of this portion of the nucleus. It is assumed that the neutrons' motion is equal and opposite to that of the protons, so that the net angular momentum is zero. At this point, no specification is made as to whether the entire nucleus or a portion only is excited, although it may later be shown that in order to get the desired results for the transverse form factor, it is necessary to assume that only an outer core is excited, with an inner core of nucleons remaining inert. ω , the frequency of the oscillation, is equal to the energy transferred to the nucleus by the interacting electron. The system of units employed in this paper is that in which $c = \hbar = 1$.

From charge conservation,

$$\bar{\nabla} \cdot \bar{J} = -\frac{\partial \rho}{\partial t}.$$

The assumed mode has

$$\bar{J} = J_{\theta} \bar{e}_{\theta},$$

where \bar{e}_{θ} is the unit vector in the θ direction. Thus, the charge

conservation equation is:

$$\frac{1}{r \sin \theta} \frac{d}{d\theta} (J \sin \theta) = -i\omega a Z' e^{-f(r)} \cos \theta e^{i\omega t}.$$

This leads to:

$$J(r) = -\frac{1}{2} i\omega a Z' e^{-f(r)} \sin \theta e^{i\omega t}.$$

The calculation of the matrix elements H_{fi} is done semi-classically.

The matrix element is [5]

$$H_{fi} = \int J_{\mu}(x) A_{\mu}(x) d^3x.$$

It is assumed that the nucleus is scattered by the electric field of the electron, which can be written as

$$A_{\mu}(x) = \frac{4\pi i e}{\underline{q} V} e^{i\underline{q} \cdot \underline{x}} \bar{u}_f \gamma_{\mu} u_i,$$

where \underline{q} is the four-momentum transfer and u_f and u_i are the final and initial plane-wave electron states [5]. V is the normalization volume for these states. It is further assumed that $J_{\mu}(x)$ for the nucleus can be written as just the classical charge and current densities for the model. Thus:

$$H_{fi} = \int J_{\mu}(x) A_{\mu}(x) d^3x = \frac{4\pi i e}{\underline{q} V} e^{i\underline{q} \cdot \underline{x}} J_{\mu}(x) \bar{u}_f \gamma_{\mu} u_i d^3x.$$

But $J_{\mu}(x)$ is just $j_{\mu}(x) e^{i\omega t}$, and

$$e^{i\underline{q} \cdot \underline{x}} = e^{i\bar{\underline{q}} \cdot \bar{\underline{x}}} e^{-i\omega t},$$

so what must be calculated is the Fourier transform of the current density:

$$j_{\mu}(\bar{q}) = \int e^{i\bar{q} \cdot \bar{x}} j_{\mu}(x) d^3x \quad .$$

3. Calculation of Fourier Transforms.

There will be three normal modes for this model: one for which the axis of vibration is parallel to \bar{q} (the longitudinal mode) and two for which the axes of vibration are perpendicular to \bar{q} (the transverse modes). For the longitudinal mode, let the z-axis denote the axis of vibration.

Then:

$$\begin{aligned} j_x(q) &= C \int r^3 f(r) \sin^2 \theta \cos \theta \cos \phi e^{iqr \cos \theta} dr d\theta d\phi \\ &= 0 \end{aligned}$$

Likewise:

$$\begin{aligned} j_y(q) &= C \int r^3 f(r) \sin^2 \theta \cos \theta \sin \phi e^{iqr \cos \theta} dr d\theta d\phi \\ &= 0 \end{aligned}$$

Thus, for the longitudinal mode, both transverse terms are zero.

However:

$$\begin{aligned} j_z(q) &= -C \int r^3 f(r) \sin^3 \theta e^{iqr \cos \theta} dr d\theta d\phi \\ &= -\frac{8\pi C}{q} \int f(r) [\sin qr - qr \cos(qr)] dr \\ &= 4\pi a Z' e \omega f_L(q) \end{aligned}$$

where

$$f_L(q) = \frac{1}{3} \int [\sin qr - qr \cos(qr)] dr .$$

The fourth component of $j_\mu(q)$ is:

$$\begin{aligned} i \int \rho(x) e^{i\bar{q} \cdot \underline{x}} d^3x &= ia \int f(r) r^2 \sin \theta \cos \theta e^{iqr \cos \theta} dr d\theta d\phi \\ &= -4\pi a Z' e q f_L(q) . \end{aligned}$$

Thus, for the longitudinal mode,

$$j_{\mu}(q) = 4\pi iaZ'e\omega f_L(q) (0, 0, 1, \frac{q}{\omega}) .$$

For the transverse modes, the calculations are done in the coordinate system in which the z-axis is the axis of vibration and the y-axis is in the direction of \bar{q} . For one of these modes:

$$j_{\rho}(q) = ia \int f(r) r^2 \sin\theta \cos\theta e^{iqr \sin\theta \sin\phi} dr d\theta d\phi = 0$$

$$\begin{aligned} j_x(q) &= C \int f(r) r^3 \sin^2\theta \cos\theta \cos\phi e^{iqr \sin\theta \sin\phi} dr d\theta d\phi \\ &= 0 \end{aligned}$$

$$\begin{aligned} j_y(q) &= C \int f(r) r^3 \sin^2\theta \cos\theta \sin\phi e^{iqr \sin\theta \sin\phi} dr d\theta d\phi \\ &= 0 \end{aligned}$$

$$\begin{aligned} j_z(q) &= -C \int f(r) r^3 \sin^3\theta e^{iqr \sin\theta \sin\phi} dr d\theta d\phi \\ &= -\frac{4\pi C}{3} \int f(r) [qr - \sin(qr)] dr = -2\pi ia\omega f_t(q) \end{aligned}$$

where

$$f_t(q) = \frac{1}{3} \int f(r) [\sin(qr) - qr] dr .$$

For the modes in which the axis of vibration is perpendicular to \bar{q} , the only nonzero Fourier transforms are those in the directions of the axes of vibration. The overall current density Fourier transform can be written:

$$j_{\mu}^P(q) = 4\pi i a \omega Z' e \left[-\frac{1}{2} \delta_{P1} f_t(q), -\frac{1}{2} \delta_{P2} f_t(q), \delta_{P3} f_t(q), \right. \\ \left. i \frac{q}{\omega} \delta_{P3} f_L(q) \right] = 4\pi i a \omega Z' e f_{\mu}^P(q)$$

where P is the polarization axis for the excited nucleus (1, 2, or 3, with the 3-axis being the axis parallel to \vec{q}) .

4. Calculation of Cross-Section.

The interaction Hamiltonian is now:

$$H_{fi}^P = \frac{4\pi e}{q^2 V} j_\mu^P(q) \bar{u}_f \gamma_\mu u_i$$

$$= -\frac{16\pi^2 a Z' e^2 \omega}{q^2 V} f_\mu^P(q) \bar{u}_f \gamma_\mu u_i$$

The cross-section depends on the square of the Hamiltonian, averaged over initial electron spin states and summed over final electron spins and nuclear polarizations. This is:

$$\overline{|H|^2} = \frac{1}{6} \sum_{i, f, P} \left(\frac{16\pi^2 a Z' e^2 \omega}{q^2 V} \right)^2 |f_\mu^P(q) \bar{u}_f \gamma_\mu u_i|^2.$$

The summations lead to a trace calculation, as follows:

$$\sum_{i, f, P} \left(f_\mu^P(q) \bar{u}_f \gamma_\mu u_i u_i^* \gamma_\nu \gamma_4 u_f f_\nu^{P*} \right) =$$

$$\text{Tr} \left[\left(\bar{u}_f \gamma_\mu u_i u_i^* \gamma_\nu \gamma_4 u_f \right) f_\nu^{P*} f_\mu^P \right].$$

Let us now examine the quantity in parentheses. If the γ_ν and γ_4 were reversed, this would just be

$$\text{Tr} \left(\bar{u}_f \gamma_\mu u_i \bar{u}_i \gamma_\nu u_f \right),$$

which is [5]:

$$- \frac{1}{EE'} [\underline{k}_\mu \underline{k}'_\nu + \underline{k}_\nu \underline{k}'_\mu - (\underline{k} \cdot \underline{k}' + m^2) \delta_{\mu\nu}] .$$

From the commutation relations for the gamma matrices:

$$\begin{aligned} \text{Tr} (\bar{u}_f \gamma_\mu u_i u_i^* \gamma_\nu \gamma_4 u_f) = \\ \frac{1}{E_i E_f} [\underline{k}_\mu \underline{k}'_\nu^* + \underline{k}_\nu \underline{k}'_\mu^* - (\underline{k} \cdot \underline{k}' + m^2) (\delta_{\mu\nu} - 2\delta_{\mu 4} \delta_{\nu 4})] . \end{aligned}$$

The form factor matrix is:

$$\begin{aligned} F_{\nu\mu} &= \frac{1}{4} f_t^2(q) \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix} + f_L^2(q) \begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & iq/\omega \\ 0 & 0 & -iq/\omega & q^2/\omega^2 \end{vmatrix} \\ &= \frac{1}{4} f_t^2(q) A_{\nu\mu}^t + f_L^2(q) A_{\nu\mu}^L . \end{aligned}$$

There are two traces which must be evaluated. These are:

The transverse coefficient:

$$\text{Tr} (B_{\nu\mu} A_{\mu\nu}^t) = 4 \sin^2(\theta/2) \left[1 - \frac{2kk'}{q^2} \cos^2(\theta/2) \right]$$

The longitudinal coefficient:

$$\text{Tr} (B_{\nu\mu} A_{\mu\nu}^L) = \frac{2q^2}{q^2} \cos^2(\theta/2)$$

where θ is the angle between the incoming electron momentum k and

the outgoing electron momentum k' . The square of the Hamiltonian, averaged and summed over states, is:

$$\overline{|H|^2} = \left(\frac{16\pi^2 a Z' e}{q^2 v} \right)^2 \omega^2 \frac{1}{12 k k'} \left\{ 2 k k' \sin^2 (\theta/2) \right. \\ \times \left[1 + \frac{2 k k'}{q^2} \cos^2 (\theta/2) \right] f_t^2 (q) \\ \left. + \frac{4 k k' q^4}{2 \omega^2} \cos^2 (\theta/2) f_L^2 (q) \right\} .$$

The amplitude a of the oscillation can be calculated (classically) from energy considerations. At the time during the oscillation such that the charge density of the nucleus is its ground-state configuration ($\omega t = (n + \frac{1}{2})\pi$), all the energy transferred to the nucleus is in the kinetic energy of the neutrons and protons. The velocity distribution of the excited protons is

$$\bar{v}(r) = \bar{j}(r) / Z' e f(r) .$$

This ratio is $v(r) = \frac{1}{2} \omega a r \sin \theta$. The mass density of the excited protons is $Z' M f(r)$. Thus, the kinetic energy distribution of the excited protons is:

$$\frac{1}{2} M Z' f(r) v^2(r) = \frac{1}{8} Z' M f(r) \omega^2 a^2 r^2 \sin^2 \theta .$$

Since half of the transferred energy will be in the protons (with the other half being in the kinetic energy of the neutrons):

$$\frac{\omega}{2} = \frac{MZ' \omega^2 a^2}{8} \int f(r) r^2 \sin^2 \theta r^2 \sin \theta d\theta dr d\phi$$

This gives

$$a^2 = \frac{3}{2\pi Z' M \omega I} ,$$

$$\text{where } I = \int f(r) r^4 dr .$$

The cross-section is, from the well-known Golden Rule of Fermi,

$$d\sigma = \sum_{\substack{\text{final} \\ \text{states}}} 2\pi |H_{fi}|^2 \delta(E_f - E_i) \frac{1}{\text{flux}}$$

The flux is just k/VE_i . The summation over final states is just an integral over d^3k' for the case at hand. The δ -function is just $\delta(k - k' - \omega)$. Thus:

$$\begin{aligned} d\sigma &= \int \frac{V}{(2\pi)^2} \frac{VE_i}{k} \overline{|H_{fi}|^2} \delta(k - k' - \omega) k'^2 dk' d\Omega \\ &= \frac{V^2 E_i k'^2}{(2\pi)^2 k} |H_{fi}|^2 d\Omega \Big|_{k - k' = \omega} \end{aligned}$$

Finally, putting $\overline{|H|}^2$ into this equation, we obtain:

$$\begin{aligned} \frac{d\sigma}{d\omega} &= \frac{\alpha^2 k'}{q^4 k} \frac{\omega}{2\pi^2 Z' M I} \left\{ 2kk' \sin^2(\theta/2) f_t^2 \left[1 + \frac{2kk'}{q^2} \cos^2(\theta/2) \right] \right. \\ &\quad \left. + \frac{4kk' q^4}{2 \omega^2} f_L^2 \cos^2(\theta/2) \right\} . \end{aligned}$$

Note that these form factors must be multiplied by a constant term

$$\frac{\omega}{2\pi^2 Z'_{MI}}$$

in order to compare them with the form factors shown in Figures 1 and 2.

5. Uniform Shell Model.

If the nuclear charge density is assumed to be uniform, with the nucleons within a shell of inner radius r_1 and outer radius r_2 being excited by the interaction, we have:

$$f_L(q) = \frac{1}{3} \left[\sin(qr_2) - qr_2 \cos(qr_2) - [\sin(qr_1) - qr_1 \cos(qr_1)] \right]$$

$$f_t(q) = \frac{1}{3} \left[\sin(qr_2) - qr_2 - [\sin(qr_1) - qr_1] \right] .$$

Figure 3 shows the results for the square of the transverse form factor versus momentum transfer for O^{16} , with the assumption that the entire nucleus is excited ($r_1 = 0$), and the same form factor with the assumption that an inner core consisting of a He^4 nucleus remains inert. The figures used for the outer radii for these calculations were 3.41 fermi for O^{16} and 2.08 fermi for He^4 [3].

If one assumes an even thinner shell is excited, the form factor is reduced by orders of magnitude and falls off to zero with increasing q even faster. Figure 4 shows the results for a shell of thickness 0.1 fermi. No matter how thin the shell, the uniform shell model cannot give the observed form factor behavior with q for a reasonable outer radius.

6. The Fermi Distribution Model.

Now let us assume that the charge density function is just the Fermi distribution:

$$f(r) = \frac{1}{1 + e^{\alpha(r-a)}} - \frac{1}{1 + e^{\alpha(r-b)}}$$

The integral can be done by breaking it up into two parts and writing each part as an infinite series, as follows:

$$f_t^d(q) \equiv \int f(r) [\sin(qr) - qr] dr = \left[\int_0^a + \int_0^\infty \right] f(r) \\ \times [\sin(qr) - qr] dr$$

For $r < a$;

$$\frac{1}{1 + e^{\alpha(r-a)}} = \sum_0^\infty (-1)^n e^{n\alpha(r-a)}$$

For $r > a$;

$$\frac{1}{1 + e^{\alpha(r-a)}} = \sum_1^\infty (-1)^{n+1} e^{-n\alpha(r-a)}.$$

The result for the form factor is:

$$f_t^a(q) = -\frac{1}{q} \left[\frac{a^2}{2} - \frac{n^2}{6\alpha^2} \right] + \frac{2 \sin^2 \left(\frac{qa}{2} \right)}{q^4} \\ - \sum_1^\infty (-1)^n \left[\frac{\frac{e^{-n\alpha a}}{2^2 \alpha^2} + \frac{2 \cos qa}{2^2 q}}{q^2 + n^2 \alpha^2} \right]$$

For excitation of a shell, the form factor is

$$f_t(q) = f_t^a(q) - f_t^b(q)$$

where a is the outer radius of the shell and b is the inner radius.

The square of the transverse form factor for 0^{16} is shown in Figure 5,

using $a = 3.64$ and these combinations for b and α : $b = 3.0$,

$\alpha = 1$; $b = 3.6$, $\alpha = 1$; and $b = 3.0$, $\alpha = 20$. Again, the thinner

the shell, the smaller the form factor and the faster it drops off with

increasing q . For large α , the square of the form factor is smaller

and does not drop off as fast. In fact, for very large α , one gets just

the uniform shell. Note that the f_t^2 curve for $\alpha = 20$ actually crosses

the $\alpha = 1$ curve. This leads one to suspect that if one assumes an outer

boundary for the excited shell with, say, $\alpha = 1$ and an inner boundary

with very large α , for a thin enough shell one could get a curve for

$|f_t|^2$ which has the desired minimum at about $q = 0.3$.

Figure 6 shows the results for three trials of this composite model.

For a thick shell ($a = 3.64$ and $b = 2.0$) with $\alpha = 1$, the form factor is

larger than for the same shell with $\alpha = 10$. The form factor becomes

larger for a thinner shell. However, for the combinations of a , b , and α

used here, the form factor does not get the desired behavior.

7. Conclusions.

None of the assumed charge distributions gives the desired behavior for the form factor. This model gives somewhat the same results as the Goldhaber - Teller model; that is, the function $|f_t(q)|^2$ is a monotonically decreasing function of q within the range of interest. Further work is necessary on the composite model before this model can be completely discarded, however. For instance, if one assumes the nucleus as a whole has a Fermi charge distribution and that the inert core is a uniform shell with a radius equal to the radius of the half-height of the Fermi distribution, what one has is essentially a classical treatment of the particle-hole theory. Figure 7 illustrates the "particle" and the "hole" associated with this model. This idea should be pursued further. There is a strong possibility that it will give results similar to those of the successful Brown particle-hole theory.

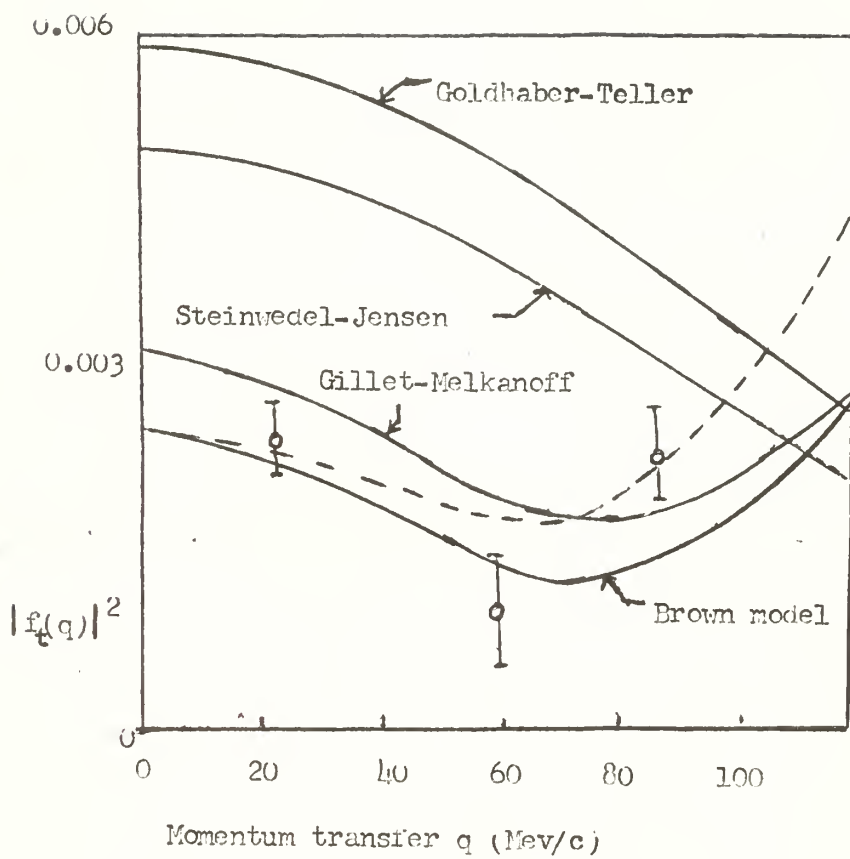


Figure 1. Square of the transverse form factor for the giant dipole resonance for Carbon 12. (Lewis and Walecka, 1964)

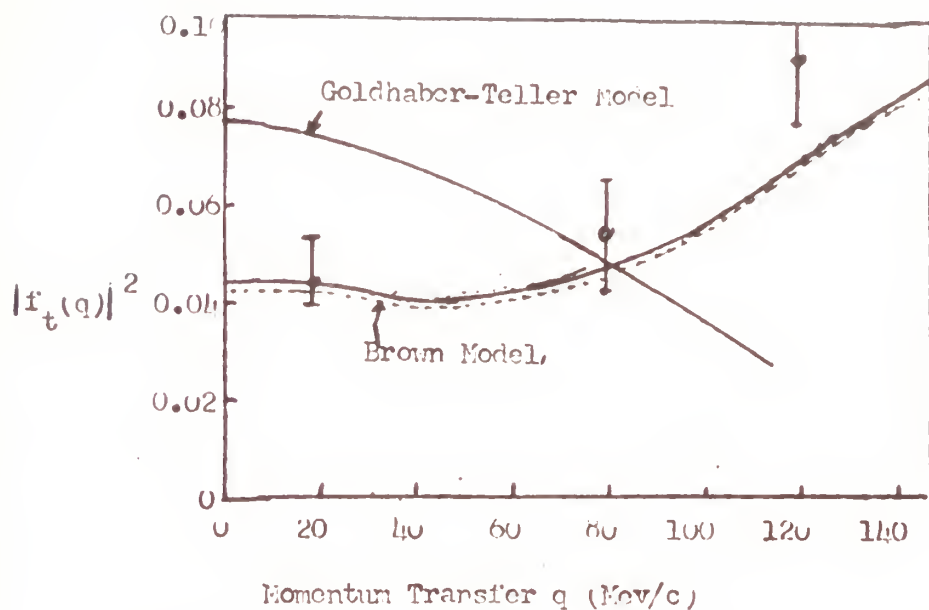


Figure 2. Square of the transverse form factor for the giant dipole resonance for Oxygen 16. (Lewis, 1963)

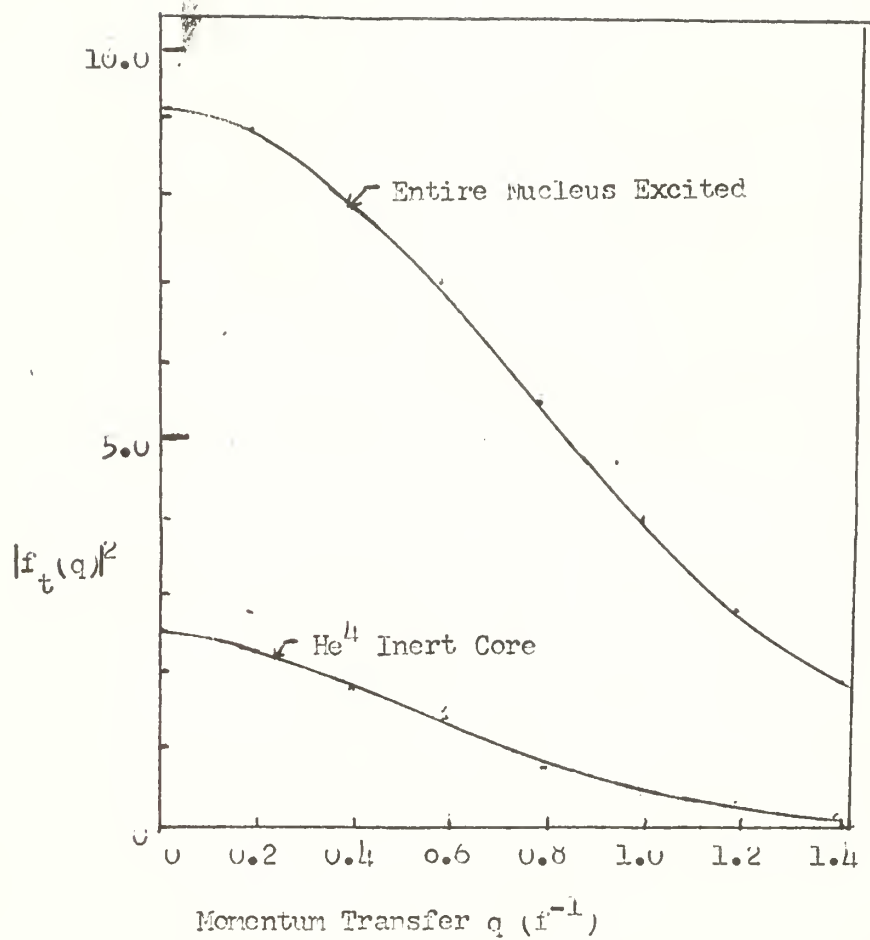


Figure 3. Square of the transverse form factor for Oxygen 16, entire nucleus excited, and outer shell only excited; uniform shell model.

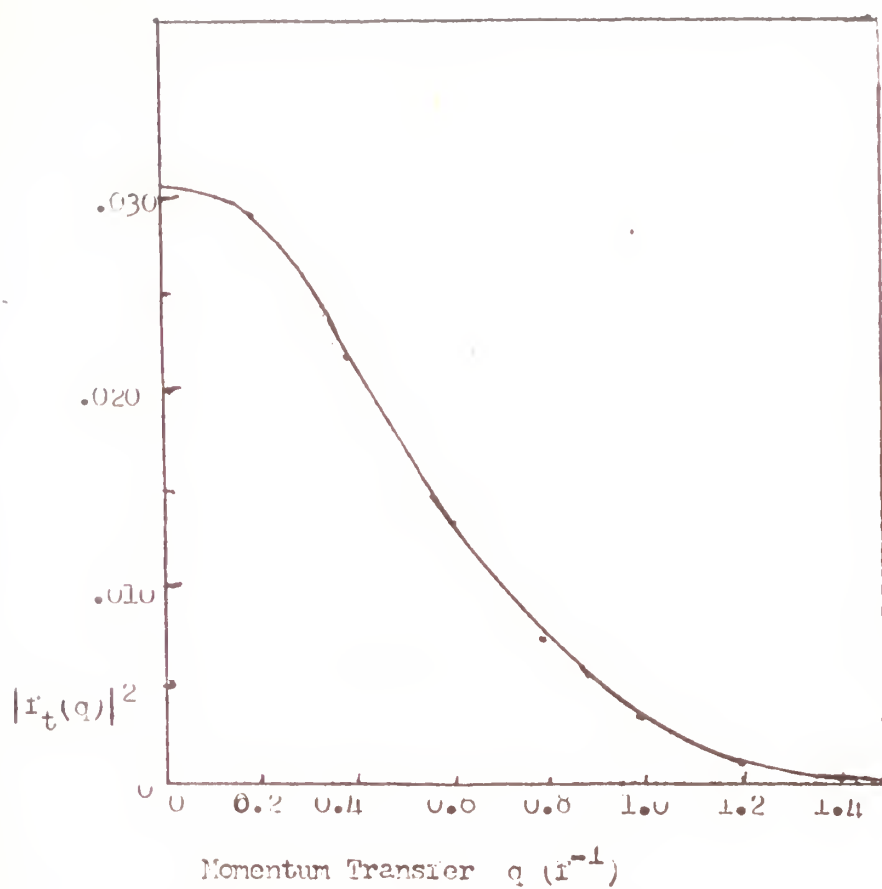


Figure 4. Square of the transverse form factor for Oxygen 16, very thin (.1 r) shell of constant density.

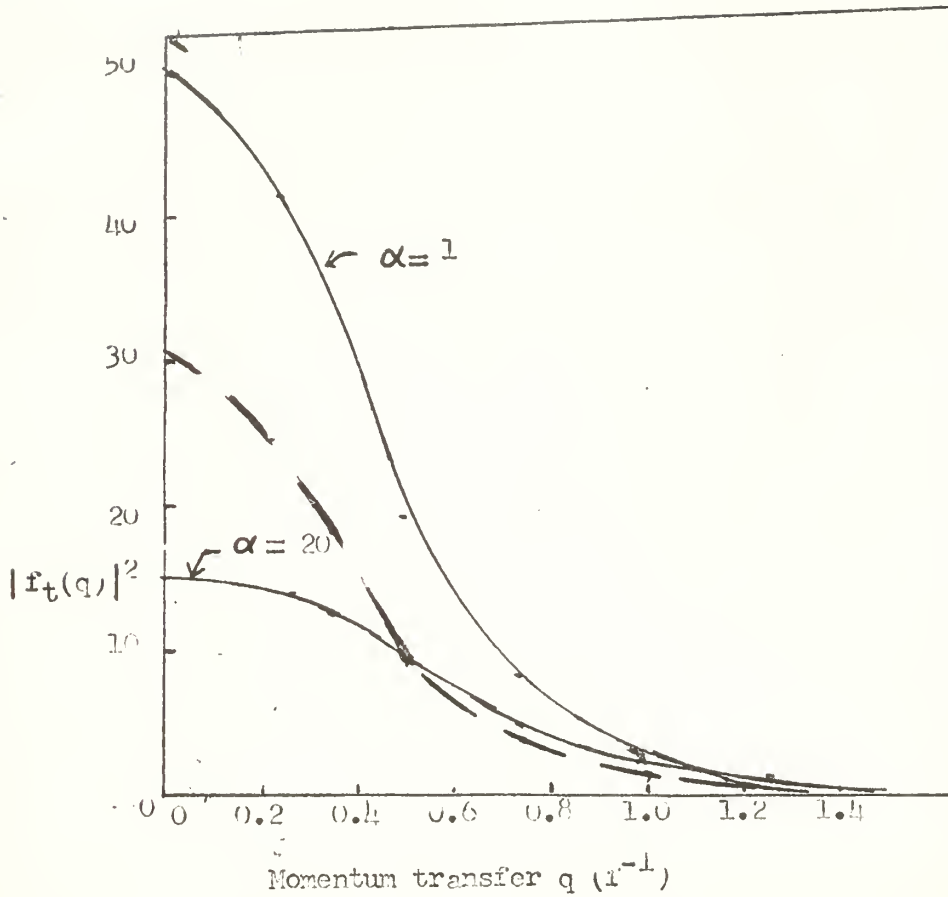


Figure 5. Square of the transverse form factor for Oxygen 16, Fermi distribution charge density, $\alpha=1$ and $\alpha=20$. The dashed line represents 100 times the square of the transverse form factor for a very thin shell (.04 fermi) with $\alpha=1$.

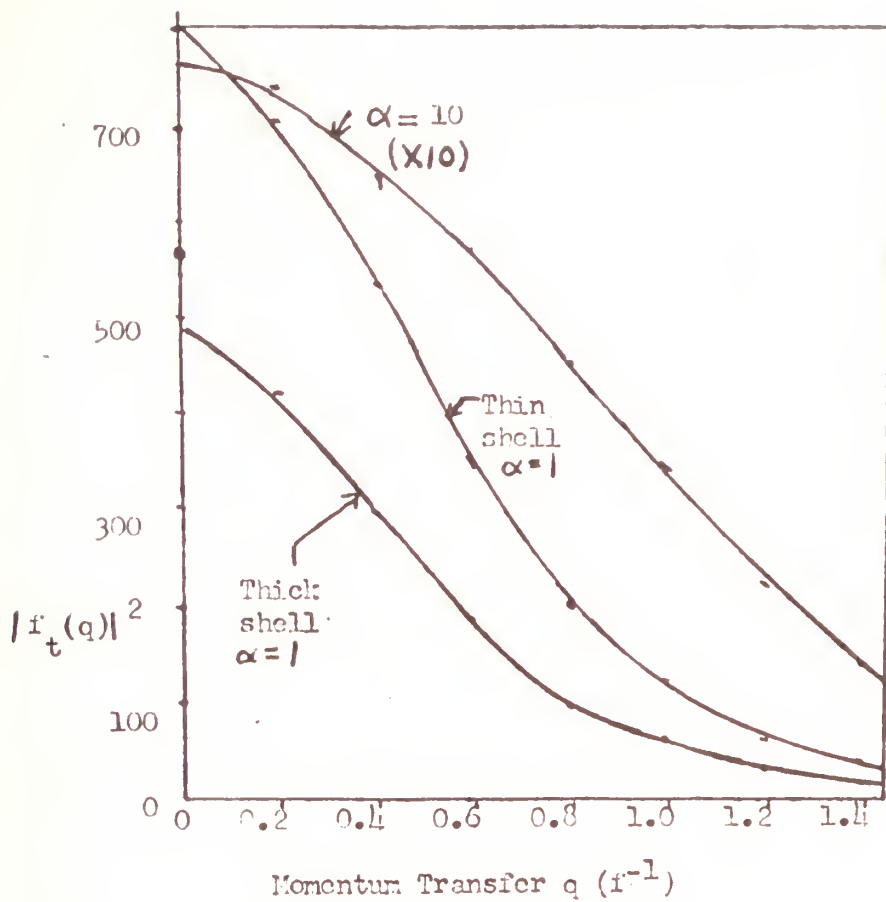


Figure 6. Square of the transverse form factor for O_{16} for the composite model.

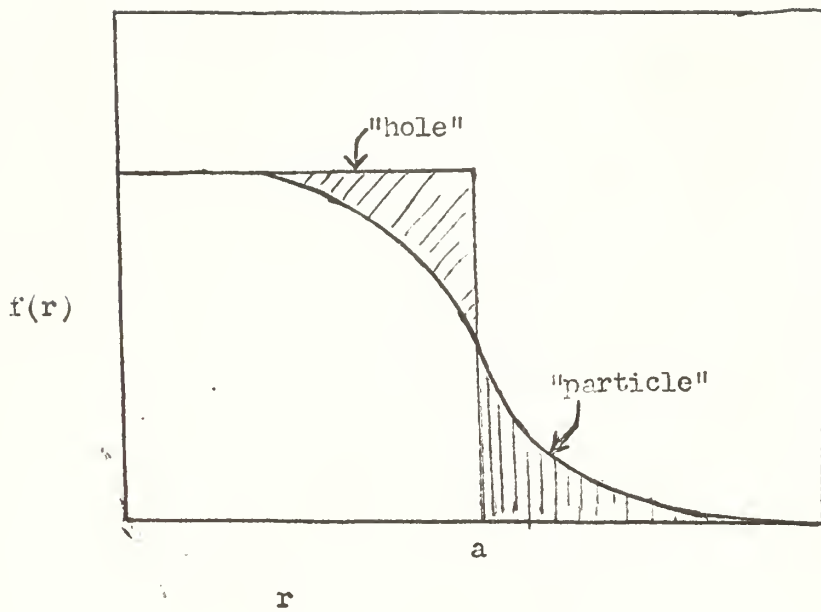


Figure 7. Classical particle-hole analogue.

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<p>The work of Lewis and Walecka on the structure of the giant resonance for electron scattering from Carbon 12 and Oxygen 16 seems to indicate that the shell model particle-hole theory gives better results than collective models -- in particular the Goldhaber - Teller and Steinwedel - Jensen models. The transverse form factor as a function of momentum transfer is observed experimentally to fall to a minimum and rise again in the range from zero to 0.6f^{-1}. This dip is predicted by the Brown particle-hole theory model but is not predicted by either of these collective models. The purpose of this paper is to develop a collective model which could possibly give this dip in the form factor. Further work is necessary to determine if this model will actually give this result. A semiclassical treatment is used to find the matrix elements for the assumed model. The transverse form factor is then calculated for two nuclear charge density functions -- constant density and the Fermi distribution.</p>			

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